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Dark matter and alternative recipes for the missing mass

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Abstract. Within the standard cosmological scenario the Universe is found to be filled by obscure components (dark matter and dark energy) for $\sim$ 95% of its energy budget. In particular, almost all the matter content in the Universe is given by dark matter, which dominates the mass budget and drives the dynamics of galaxies and clusters of galaxies. Unfortunately, dark matter and dark energy have not been detected and no direct or indirected observations have allowed to prove their existence and amount. For this reason, some authors have suggested that a modification of Einstein Relativity or the change of the Newton's dynamics law (within a relativistic and classical framework, respectively) could allow to replace these unobserved components. We will start discussing the role of dark matter in the early-type galaxies, mainly in their central regions, investigating how its content changes as a function of the mass and the size of each galaxy and few considerations about the stellar Initial mass function have been made. In the second part of the paper we have described, as examples, some ways to overcome the dark matter hypothesis, by fitting to the observations the modified dynamics coming out from general relativistic extended theories and the MOdyfied Newtonian dynamics (MOND).

1. Introduction

Within the standard cosmological framework, the so called Λ cold dark matter (ΛCDM) scenario, the Universe is predicted to be filled by a huge amount ($\sim$ 95%) of obscure energy components (dark matter, DM, and dark energy, DE), and only $\sim$ 5% is made by baryons (i.e. stars and gas). According to several observations (i.e. distance moduli of high-z Supernovae Ia, anisotropies in the cosmic microwave background radiation, etc.) the Universe is spatially flat and in an accelerated phase of expansion. DE amounts to $\sim$ 75% of the total energy budget, is modelled as a negative equation of state and is the main driver of the observed cosmic acceleration [1, 2, 3]. DM has an important role not only at the cosmic scale but also to explain the growth of the structures and the evolution of galaxies across the cosmic time. Although this is the widest accepted theory to explain the various astronomical and cosmological observations, there has been no direct observational evidence for both DM and DE. Anyway, this has motivated the investigation of alternative theories to explain observations without invoking the presence of such unknown ingredients. We will first concentrate on DM in galaxies (mainly massive ellipticals) and on some alternative recipe. In particular we will briefly discuss the viability of two approaches: 1) a relativistic one consisting in a generalization of the Einstein' equations, the so called $f(R)$ theories, and 2) a classical one, by means of a modification of the second Newton's law, as the MOdified Newtonian Dynamics (MOND).

2. The missing mass

The need of DM in galaxies and clusters was firstly suggested by Fritz Zwicky in 1934 to account for evidence of "missing mass" in the observed orbital velocities of galaxies in clusters [4]. Applying the virial theorem to the Coma cluster he found a total mass 400 times larger than that visually observed. Till 60s, 70s no other observational evidences for the existence of DM were found. However, it was only later, in 70s, that the evidences started to be clear enough to put the DM at the center of the astronomical community attention. In these years, some authors observed that the stars and gas in spiral galaxies have an orbital velocity which remains constant even beyond their optical radii (i.e., beyond the region where much of the stars are located) [5, 6]. Nowadays, many observations have pointed out the existence of this missing mass problem in many spirals and other kinds of galaxies, as low surface brightness and ellipticals, and clusters [7, 8, 9]. The mass distribution within these different astrophysical objects is investigated by means of different mass probes. In particular, rather than circular velocity in spirals, strong (and weak) gravitational lensing, velocity dispersion measurements and X-ray observations are powerful mass tracers inside ellipticals and clusters of galaxies.

Lately, important progresses have been made also in the study of the mass distribution in elliptical galaxies, mainly thanks to the development of the discrete velocity mapping with globular clusters (GCs; [10]) and in particular the doppler measurement of the OIII emission from Planetary Nebulae [11, 12, 13, 14, 15, 16]. PNe in particular (being stars) have allowed the stellar kinematics in the (otherwise inaccessible) outer regions of such galaxies to be studied [17].

3. Dark matter

Within the Einstein framework, the missing mass needs to be filled by DM, which seems to be dominant in the very external regions of galaxies, where the stars are absent, and also in the central regions, depending on the stellar ingredients which have been adopted. Although in the central regions many uncertainties in the DM content determination arise from the contribution of stars, many kinds of observations probing the mass in these regions and for huge samples of galaxies are available. In Tortora et al. 2009 [19] and Napolitano, Romanowsky & Tortora 2010 [20], we have recently investigated the DM content within 1 $R_{\rm eff}$ (i.e., the radius where is enclosed one-half of the total projected light in the galaxy) for a sample of local ellipticals from the sample in Prugniel & Simien 1996 [18]. The stellar mass, M_*, is determined by fitting synthetic spectral models to the measured colours, while dynamical (total) mass, $M_{\rm dyn}$ within 1 $R_{\rm eff}$, is recovered by adopting a singular isothermal sphere (SIS) for the total mass distribution[1] and the observed central velocity dispersion is used to infer the best mass profile (see [19] for further details).

Djorgovski & Davis 1987 [21] and Dressler et al. 1987 [22] discovered a tight relation among central velocity dispersion σ_0, the effective radius $R_{\rm eff}$, and effective surface brightness $I_{\rm eff}$, the so-called Fundamental Plane (FP). It has been observed a deviation of its coefficients from the virial theorem expectation under the assumptions of homology and constant mass-to-light ratio (M/L). One possible explanation of the tilt is a variation of the total M/L with galaxy luminosity/mass [22], often parameterized as a power law, $M/L \propto L^{\gamma}$. This relation might reflect a variation of different galaxy properties with mass. Thus, we have evaluated the impact on γ of various factors, like stellar population properties (metallicity, age and SF history), IMF,

[1] An isothermal profile has been shown to reproduce quite well the total mass profile in massive ellipticals [28, 29]

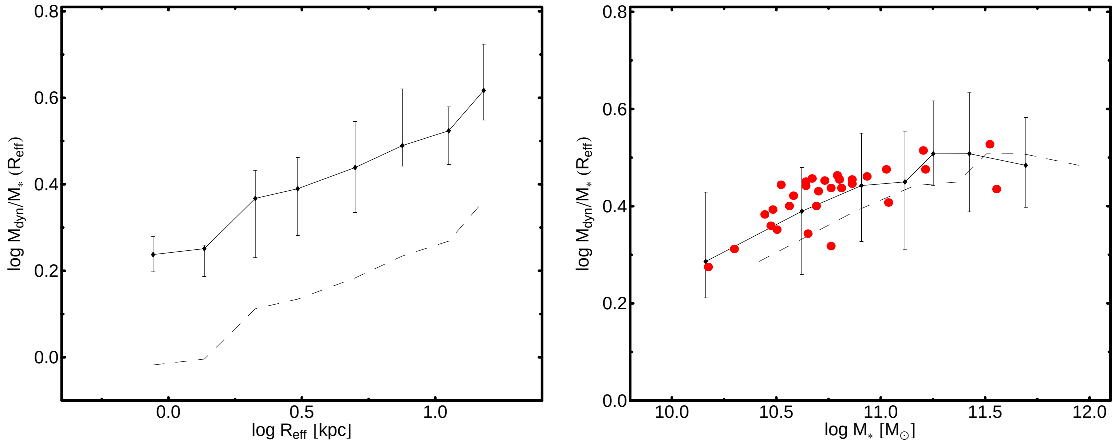

Figure 1. Dynamical-to-stellar mass ratio as a function of $R_{\rm eff}$ (left) and M_* (right) for the pure ellipticals from Prugniel & Simien 1996 [18]. Continue lines and bars are for medians and 25-75th quantiles when a Chabrier IMF is adopted, while the dashed lines are the median trends for a Salpeter IMF. The red points in the right plot are the results for simulated central galaxies in Ruszkowski & Springel 2009 [34].

rotational support, luminosity profile, non-homology and DM fraction. We found that the stellar M/L contributes little to the tilt, suggesting that the tilt is ascribed to DM.

We surveyed trends of the DM fraction within 1 $R_{\rm eff}$, finding it to be increasing with luminosity and stellar mass. Our conclusions are sensitive to various systematic uncertainties which we have investigated in detail, but are consistent with the results of dynamics studies at larger radii [19]. We also find that stronger correlations are found in terms of central velocity dispersion and effective radii, which means that at larger size, we go deeper in the halo, finding more DM dominated regions [20]. In particular, in Fig. 1 we show the deprojected dynamical-to-stellar ratio (adopting a Chabrier IMF [33]) as a function of $R_{\rm eff}$ and M_* for the pure elliptical galaxies in our sample. We find that $M_{\rm dyn}/M_* \propto {R_{\rm eff}}^{0.27}$ and $M_{\rm dyn}/M_* \propto {M_*}^{0.13}$, this last being steeper than the slope of ~ 0.06 found in Hyde & Bernardi 2009 [26]. In the smallest and less massive galaxies is $M_{\rm dyn}/M_* \sim 1.5$, i.e. DM is $\sim 40\%$ of the total mass, while in the largest and most massive galaxies is $M_{\rm dyn}/M_* \sim 3-4$, corresponding to a DM fraction of $\sim 70\%$. A Salpeter IMF [31] produces larger stellar masses, and thus lower $M_{\rm dyn}/M_*$. A rigid shift is observed in the $M_{\rm dyn}/M_* - R_{\rm eff}$ plot, null DM fractions are found in the smaller galaxies and $\sim 50\%$ of DM in the largest systems. We also compare our results with the simulated galaxies in Ruszkowski & Springel 2009 [34], finding a very good agreement in the average values, but the scatter in the simulated data is smaller. We have confirmed the increasing trend with mass and $R_{\rm eff}$ adopting a sample of intermediate redshift lens galaxies from SLACS survey [30] with available lensing and dynamical information, varying the stellar recipe and the galaxy model in [23, 24, 25].

In [20] we have also examined the correlations between masses, sizes and SF histories. We confirmed an anticorrelation between $R_{\rm eff}$ and stellar age and went on to survey for trends with the central content of DM. An average relation between the central DM density and galaxy size of $< \rho_{\rm DM} > \propto {R_{\rm eff}}^{-2}$ provides the first clear indication of cuspy DM haloes in these galaxies. From the comparison with ΛCDM expectations (see details in [20, 27]), our data are found to be consistent with a Chabrier or Kroupa IMF if the halo is adiabatically contracted[2], while,

[2] Within ΛCDM, (DM only) N-body simulations predict that galaxy and galaxy cluster halos follow a universal

if we want to retain the original outcomes from N-body simulations (i.e. the NFW profile), a Salpeter IMF is best suited (in agreement with [37]). The DM density scales with galaxy mass as expected, deviating from a universal halo profile for dwarf and spiral galaxies (see also [24, 25]).

The IMF inferred from star counts in local spiral galaxies is bottom-light (as a Chabrier or Kroupa [32]), but no direct observations are possible for other galaxy types (as, e.g., ellipticals), different environments and at higher redshift. Therefore, the IMF is a fundamental ingredient which has still to be fully understood. Anyway, Chabrier and Salpeter IMFs are assumed as the limiting cases, producing a difference in stellar masses of 0.25 dex and we have shown, that also assuming a Salpeter IMF a residual DM fraction is surviving. In order to have, on average, $M_{\rm dyn} = M_*$ we need a IMF producing stellar M/L ratios $\sim 2.7^{+1.4}_{-0.9}$ ($1\,\sigma$ uncertainty) times the ones from a Chabrier IMF. For only $\sim 20\%$ of the galaxies in our sample a Salpeter IMF is totally filling the gap, avoiding the need of DM (mainly in the smallest galaxies, see left panel in Fig. 1), but for many of the galaxies larger stellar M/L would be needed. Such very high stellar masses could be produced by 1) a power-law IMF with a slope steeper than the Salpeter one[3] [38] and equivalently 2) a “bottom-heavier” and/or a “top-lighter” IMF. But, no indication exists in favour of such IMF alternatives in the local universe and they seem quite unrealistic, suggesting that DM is still an unavoidable ingredient. Anyway, recently van Dokkum & Conroy [39], analyzing the spectra of massive ellipticals in the Virgo and Coma clusters, have found that low mass stars are very abundant (contributing to $\sim 60\%$ of the total stellar mass) in such galaxies and point to a steeper IMF, which in according to the comments above, would fill the gap, without needing DM, at least in the central regions.

Finally, we introduced a new fundamental constraint on galaxy formation by finding that the central DM fraction decreases with stellar age. This result is only partially explained by the size-age dependencies, and the residual trend goes on the opposite direction to basic DM halo expectations. Therefore, we suggested that there may be a connection between age and halo contraction or IMF and that galaxies forming earlier had stronger baryonic feedback, which expanded their haloes, or lumpier baryonic accretion, which avoided halo contraction, or a lighter IMF. Using the intermediate-redshift galaxy sample from the SLACS survey, we have a confirmation of such results and a negligible evidence of galaxy evolution over the last ~ 2.5 Gyr other than passive stellar aging [27].

4. MOdified Newtonian gravity

The results discussed above have been obtained assuming that the Einstein relativistic theory, and the classical Newtonian theory of gravity as its limit, may be used also on galactic scales. However, the outer regions of galaxies typically are in a low acceleration regime, and in this regime Newtonian dynamics has never been experimentally tested. Motivated by this consideration, Milgrom (1983) [40] proposed to modify Newton’s second law of dynamics as $F = mg$, where the acceleration g is now related to the Newtonian one $g_{\rm N}$ as $g\,\mu(g/a_0) = g_{\rm N}$. The theory thus obtained is referred to as MOND. Here, a_0 is a new universal constant and $\mu(x)$ may be an arbitrary function with the properties $\mu(x >> 1) = 1$ and $\mu(x << 1) = x$, i.e. Newton’s law is recovered in high acceleration regimes, while at extremely low accelerations we have the deep-MOND regime, i.e. $F \propto g^2$. Since at large distance from the center is $\mu(g/a_0) = g/a_0$, the relation $m\mu(g/a_0)g = GMm/r^2$ becomes $g^2/a_0 = GM/r^2$ or $g = \sqrt{GMa_0}/r$. Using the relation between acceleration and velocity in a circular orbit we find $v = \sqrt[4]{GMa_0}$. Thus, MOND reproduces observed rotation curves in spiral galaxies (e.g. [41, 42]), gives a theoretical

mass density profile, the so called Navarro, Frenk & White (NFW) profile, [35]. While gas is falling down in the DM potential well, stars form and drag in the centers some DM, adiabatically contracting the DM distribution [36].

[3] In Tortora et al. 2009 [19] we have shown that if a power-law IMF with a slope of $x = -1.85$ is adopted, then the stellar M/L ratio are ~ 3.2 times the ones from a Chabrier IMF.

interpretation of the empirically determined Tully-Fisher law [41], and also works in dwarf spheroidals [43]. On the contrary, clusters seem to need extra DM to accommodate MOND [44], although recently it has been shown that sterile neutrinos can fill the gap [45].

We have considered a sample of $\sim$ 9000 local ellipticals from the SDSS sample [46] and a Chabrier IMF is adopted. For the first time we have shown that MOND is able to reproduce the dynamics in the central regions (typically $\lesssim R_{\rm eff}$) of a large sample of local ellipticals. The boost to the velocity dispersion in the MOND scenario helps to reduce the need of both radial anisotropy (in the Newtonian case without DM), deviations from a Chabrier IMF or DM. We also showed that MOND is able to predict a FP for ellipticals (similarly to the Newtonian case), but a tilt between the observed and the MOND FP is found [47]. We are working to study if MOND is able to reproduce the dynamics of both ellipticals and spirals, investigating more carefully the role of anisotropy and of the IMF.

5. Extended theories of gravity

An alternative approach to the missing mass problem consists to modify the Einstein's equations, 1) or by introducing a field (with non zero mass) which is coupled with matter, as in the so called tensor-vector-scalar (TeVeS) theories [48, 49, 50], or 2) by replacing the Ricci tensor R in the Einstein-Hilbert action with a function of R, $f(R)$. Although the TeVeS theories have been shown to be successful to reproduce different observations (e.g. [49]), we will concentrate on the $f(R)$ theories, which have been suggested to reproduce both DE on cosmic scales [51] and DM [52, 53] (see also [54, 55, 56] for a complete review on the subject).

It has been demonstrated that very general $f(R)$ analytical theories can induce a modification in the dynamics of massive particles. In particular, if one solves the field equations in the weak field limit under the general assumption of an analytic Taylor expandable $f(R)$ functions of the form

$$f(R) = f_0 + f_1 R + f_2 R^2 + ... \tag{1}$$

the new gravitational potential can be written as

$$\phi(r) = -\frac{GM}{(1+\delta)r}\left(1 + \delta e^{-\frac{r}{L}}\right) \tag{2}$$

where the first term is the Newtonian-like part of the potential associated to baryonic point-like mass $M/(1+\delta)$ (no DM) and the second term, the Yukawa-like potential, is a modification of the gravity including a scale length, L, associated to the above coefficients of the Taylor expansion. This gravitational potential was also adopted by Sanders 1984 [57], under the assumption of anti-gravity generated by massive particles (the so called Finite Length-scale Anti-Gravity, FLAG, theory). This theory conjectures that in addition to the massless graviton, particles with mass can carry the gravitational force, an anti-gravity force (if $-1 < \delta < 0$), which can allow to solve the missing mass problem. We have performed the first analysis of extended stellar kinematics (up to 7 $R_{\rm eff}$s) of 3 ellipticals (NGC 3379, NGC 4374, NGC 4494), where the Yukawa-like gravitational potential in Eq. (2) is considered [58]. We find that these modified potentials are able to fit quite well all galaxies in our sample and the orbital anisotropy distribution turns out to be similar to the one estimated if a dark halo is considered. The parameter which measures the strength of the Yukawa-like correction, δ ($= -0.88, -0.79, -0.75$ for the three galaxies) is, on average, larger than the one found previously in spiral galaxies ([57], $-0.95 \lesssim \delta \lesssim -0.92$) and seemingly correlating with the orbital anisotropy.

Though the additional Yukawa term in the gravitational potential modifies dynamics with respect to the standard Newtonian limit of General Relativity, we have demonstrated that the motion of massless particles results unaffected thanks to suitable cancellations in the post-Newtonian limit [59]. Thus, all the lensing observables are equal to the ones known from General

Relativity. We are planning to test the implications of these results by fitting both lensing and dynamical observables in galaxy and cluster lensing events. In particular, following [27], the sample of lens galaxies at intermediate redshift from SLACS sample will be an efficient testing ground for these extended theories.

6. Conclusions

In this contribution we have discussed the missing mass problem, which come out from different observations in galaxies and clusters. In a standard Newtonian and Einstein approach, DM is an unavoidable ingredients to explain both cosmological observations and dynamics in galaxies and clusters. DM is found to be the dominant component, at all, since the stellar content is negligible with respect to the total mass determined by, e.g. dynamical analysis. Less obvious is the DM role and its amount in the central regions (typically $\lesssim R_{\rm eff}$) where a very large fraction of stars are settled. We have shown that DM in the central regions is strongly dependent on the IMF assumption. In fact, while the typical "bottom-light" (Chabrier or Kroupa) IMFs, found through direct observations in local spirals, predict lower stellar masses and thus larger DM fractions, $\sim 40-70\%$ in massive ellipticals (with stellar mass $M_* \gtrsim 10^{10} M_\odot$), a Salpeter IMF predict larger stellar masses, and thus lower DM fractions $\sim 0-50\%$. We have also shown how the typical halo density profiles derived from ΛCDM N-body simulations are consistent with the only if stars are distributed in according to a Salpeter IMF, while a contraction of the DM halo is needed to reconcile a Chabrier IMF. Moreover, to avoid any DM in the central regions, we would need steeper IMFs, but no strong observation seems now convincing us which this is the case (e.g. [39]). The situation in the external regions (few $R_{\rm eff}$s up to the virial radius) is more complicated, and such a steep IMF seems to us not enough to replace the amount of missing mass needing in such external regions.

Anyway, until no direct indication of the existence of DM will be provided by the many working (or future) experiments, people are encouraged to search for some alternative theories to overcome the need of such unseen matter source. This can be done by means of different approaches, within both a relativistic theory, as an extension of the General Relativity, and within a classical Newtonian approach, like in the MOND. We have shown as both MOND or $f(R)$ theories are able to reproduce a wide set of observations, in particular, the dynamics in different kinds of galaxies, and recently both central ($\lesssim R_{\rm eff}$) and more external (few $R_{\rm eff}$) dynamical data in ellipticals [47, 58].

In the future we expect to investigate the correlations between DM content, mass, IMF and stellar population parameters, checking what is the impact on such correlations when the gravity theory is changed. Such theories need to be tested on wider samples of (different type) galaxies, and galaxy clusters, relying on different kinds of observations (gravitational lensing, dynamics, X-ray, etc.), in order to check if DM can be avoided at all, everywhere in galaxies or if our constrain on DM in galaxies have to be revised downward. To refuse/validate any of such theories, as alternatives to DM, we need a) to fit data quite good, at least as much as in the DM case [58], b) that some parameters derived from the fitting procedure (as other model parameters, velocity dispersion anisotropy, stellar mass-to-light ratio if allowed to be free, etc.) assume physically reasonable values [47, 58] and c) different observational probes give consistent results. If such points would be addressed, then the theory analyzed can be considered as a viable alternative. Anyway, a definite answer about the existence of DM and its abundance will arrive from both direct and indirect experiments in progress and scheduled to start in the few next years.

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